Analysis of Competition Games among (Wireless) Operators

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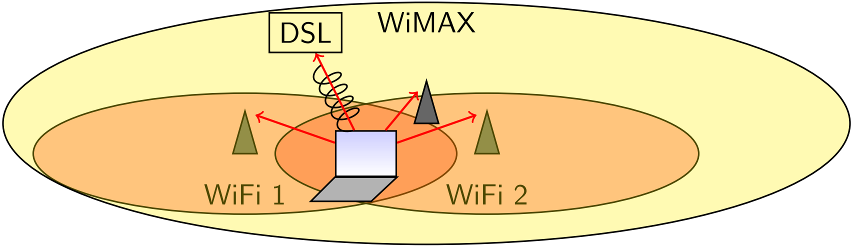
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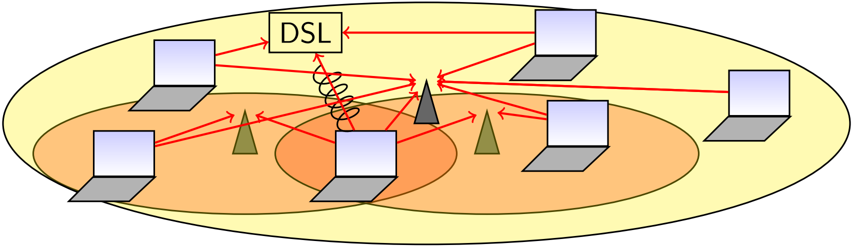
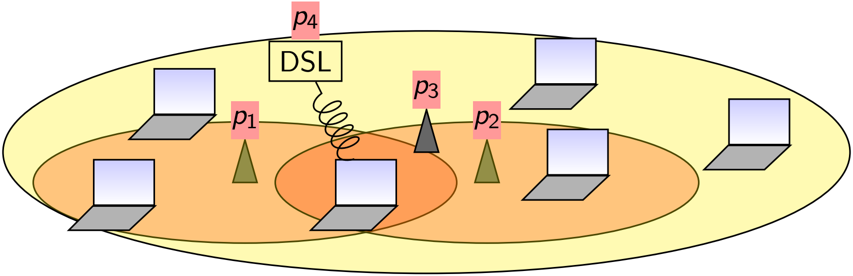


- Interactions among non-cooperative consumers: *game*
- Congested networks provide poorer quality (packet losses)

But **providers** play first!



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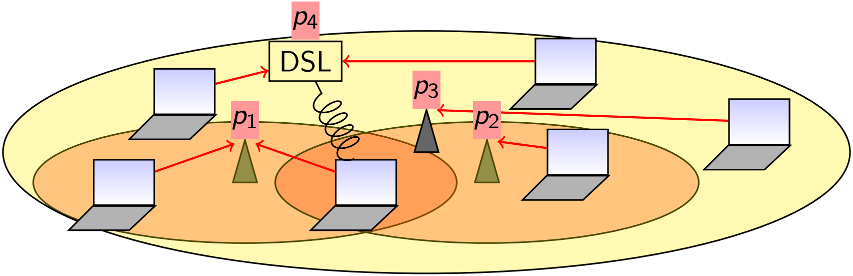


This work: study of the two-level noncooperative game.

1. *Higher level*: **providers** set prices to maximize revenue
2. *Lower level*: **consumers** choose their provider

Related work

Many references on network pricing, with different objectives:

- control congestion, Key & Massoulié'99, Lazar & Semret'99
- ensure fairness, Kelly *et al.*'98, Marbach'02
- manage different QoS levels, Cocchi *et al.*'93, Odlyzko'99
- maximize network revenue. Paschalidis & Tsitsiklis'00

But only few considering competition among providers:

- wireless providers playing on trans. power Felegyhazi & Hubaux'06
- studies of peering agreements He & Walrand'03'05 Shakkotai & Srikant'05
- competition with delay-sensitive users Acemoglu & Ozdaglar'06 Hayrapetyan *et al.*'06

This work: competition among providers with **loss-sensitive users** and **minimal regulation** $\Rightarrow$ performance of the outcome?

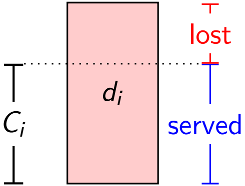
Outline

Outline

1. The pricing and competition model
   - Charged and perceived prices
   - Lower level game: user choices
   - Higher level game: price war
2. Model 1: common coverage area
   - User equilibrium
   - Price war outcome
3. Model 2: two providers with imbricated coverage areas
   - User equilibrium and price war
   - Optimizing the transmission power
4. Model 3: partial spectrum sharing
   - User choices and provider prices equilibria
   - License or share?
5. Conclusions and perspectives

Communication model: packet losses

- Time is slotted
- Each provider i has finite capacity C_i
- If total demand d_i at provider i exceeds C_i: exceeding packets are *randomly* lost



$$\mathbb{P}(\text{successful transmission}) = \min\left(1, \frac{C_i}{d_i}\right)$$

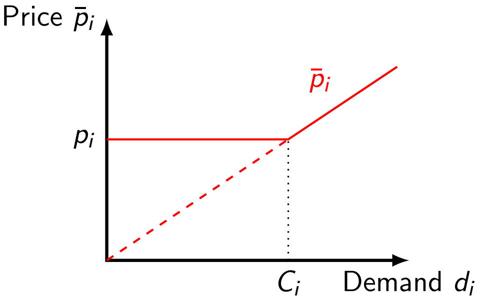
$$\Rightarrow \text{Expected number of transmissions} = \frac{1}{\mathbb{P}(\text{success})} = \max\left(1, \frac{d_i}{C_i}\right)$$

Only “regulation”: pay for what you send

The price p_i at each provider i is per packet *sent* — Marbach'02

$\Rightarrow$ If several transmissions are needed, the user pays several times

$$\bar{p}_i := \textit{perceived}\text{ price at } i = \mathbb{E}[\text{price per packet}] = p_i \max\left(1, \frac{d_i}{C_i}\right)$$



Model for user choices: Wardrop equilibrium

- Users choose the provider(s) i with lowest $\bar{p}_i = p_i \max\left(1, \frac{d_i}{C_i}\right)$

$\Rightarrow$ For a given coverage zone Z, all providers with customers from that zone end up with the same perceived price $\bar{p}_i = \bar{p}_z$ Wardrop'52

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- The total demand level in a zone z depends on that price:

$$d_z = \alpha_z D(\bar{p}_z), \quad i.e. \quad \bar{p}_z = \underbrace{v}_{\text{marg. val. function}}\left(\frac{\sum d_{i,z}}{\alpha_z}\right)$$

with D the total demand function, α_z the population proportion in zone z, and $d_{i,z}$ the demand in zone z for provider i.

Higher level: price competition game

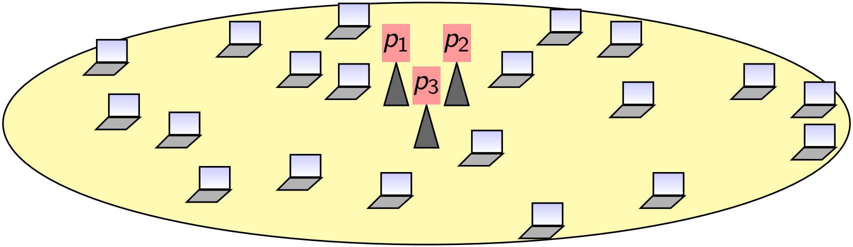
- Providers set their price p_i *anticipating users reaction*
  $\Rightarrow$ Providers are Stackelberg leaders
- We can assume management costs of the form $\underbrace{\ell_i(d_i)}_{\text{nondecreasing, convex}}$

  Provider i's objective: $R_i := p_i d_i - \ell_i(d_i)$.

Outline

Competition model

- Simplified topology: common coverage area
- N competing providers declaring price and capacity ($\mathcal{I} := \{1, \ldots, N\}$)



User equilibrium

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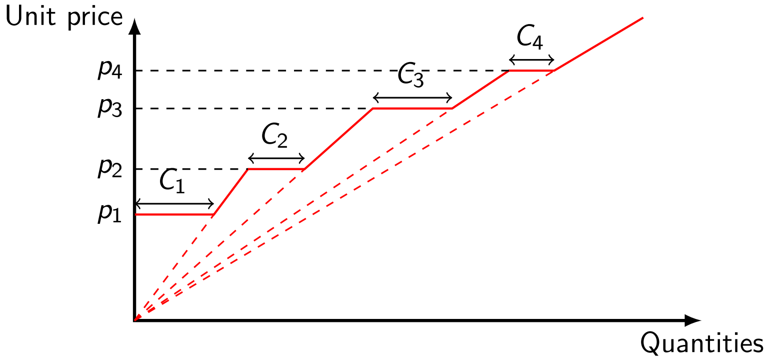

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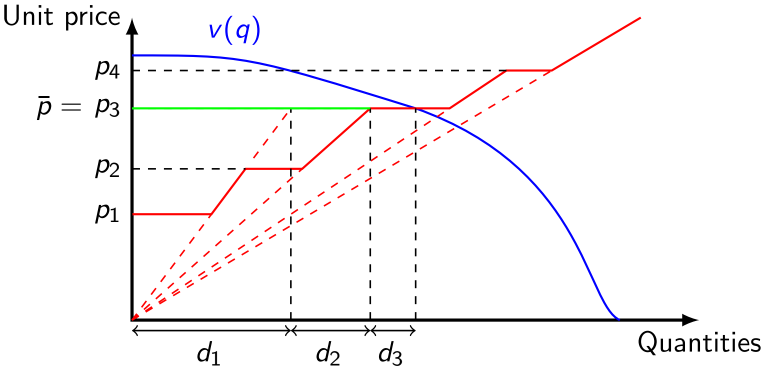

Price competition, main result

Proposition

Under condition (1) on management cost functions ℓ_i, there exists a **unique Nash equilibrium** *on price war among providers, given by*

$$\forall i \in \mathcal{I}, \quad \begin{cases} p_i &= v\left(\sum_{j\in\mathcal{I}} C_j\right) \\ d_i &= C_i. \end{cases}$$

- *Sufficient condition:* For each provider i,

$$\ell_i'(C_i) \leq \left(1 - \frac{C_i}{\sum_{j\neq i} C_j}\right) v\left(\sum_i C_i\right). \qquad (1)$$

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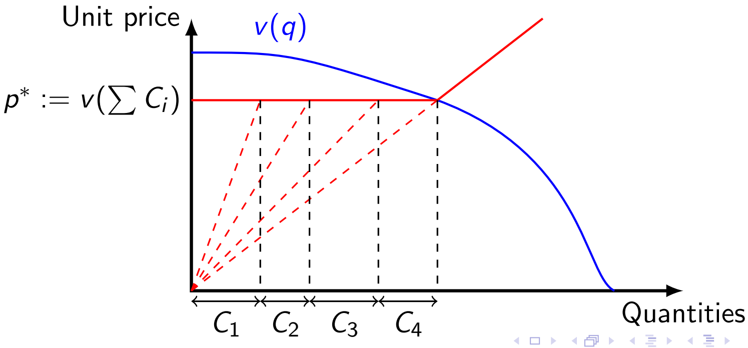


Social Welfare considerations

- A performance measure of the outcome $(d_1, ..., d_I)$ of the game $=$ overall value of the system

$$\text{Social Welfare} := \underbrace{\int_0^{\text{Throughput}} v}_{\text{users willingness-to-pay}} - \sum_i \ell_i(d_i),$$

with Throughput $:= \sum_i \min(d_i, C_i)$.

- **Remark:** under (1), the Social Welfare maximization problem leads to the same outcome $d_i = C_i \quad \forall i$ as the price war.
- **Consequence:** The Nash equilibrium corresponds to the socially optimal situation: the Price of Anarchy is 1!.

Game on declared capacities: a third level

We now consider a 3-stage game:

1. Providers $i \in \mathcal{I}$ declare their capacity C_i
2. Providers fix their selling price p_i
3. Users select their providers

Opposite effects of lowering one's capacity:

- the unit selling price at equilibrium increases and the managing cost decreases because the quantity sold decreases
- whereas on the other hand less quantity sold means less revenue.

Proposition

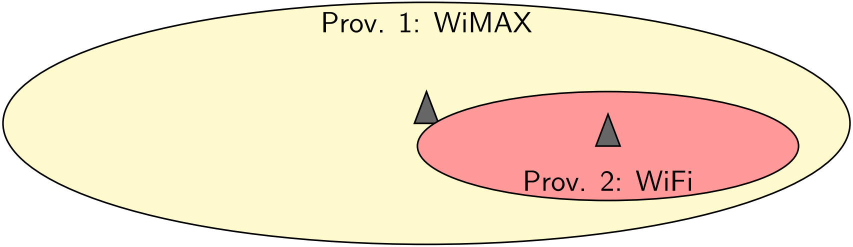
Under (1), if **demand elasticity** $\frac{-pD'(p)}{D(p)}$ *is larger than* 1*, then no provider can increase its revenue by artificially lowering its capacity* ($D \equiv v^{-1}$).

Outline

Competition model

Assumptions

- Two competing providers declaring price and capacity
- One coverage area included in the other



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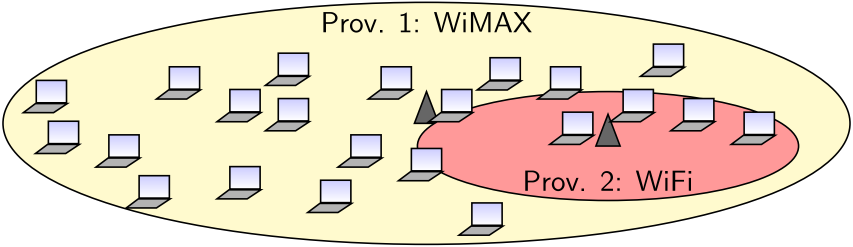
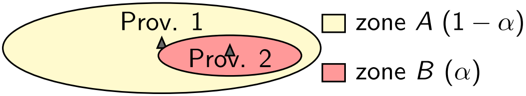
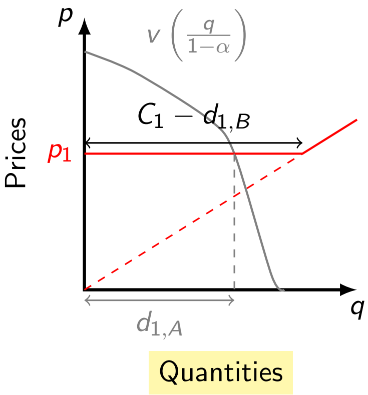
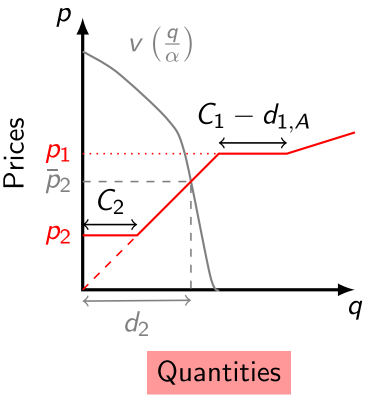


User equilibrium: illustration







User equilibrium: mathematical formulation

For each zone z and each provider i, j, at user equilibrium

$$\begin{aligned} \bar{p}_i &= p_i \max\left(1, \frac{d_i}{C_i}\right) \\ d_z &= \alpha_z D\left(\min_{i \in z} \bar{p}_i\right) \end{aligned}$$

If $i, j \in z$, then $\bar{p}_i > \bar{p}_j \Rightarrow d_{i,z} = 0.$

User equilibrium: existence and uniqueness

Proposition

For all price profile, there exists at least a user (Wardrop) equilibrium. Moreover, the corresponding perceived prices of each provider are unique.

NB: demand repartition among providers is not necessarily unique.

Higher level: price competition game

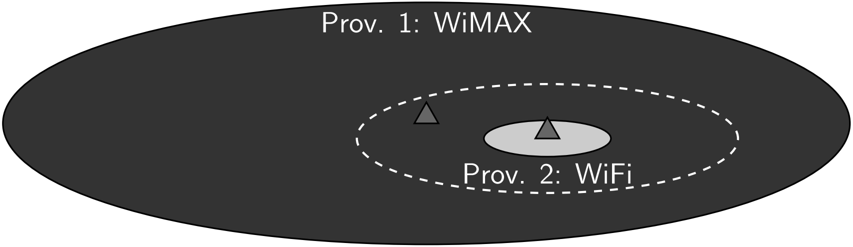
- Provider i's objective: $R_i := p_i d_i$ (no management costs).

Proposition

If $-\frac{D'(p)p}{D(p)} > 1, \quad \forall p$ *(elastic demand), then there exists a unique Nash equilibrium* (p_1^*, p_2^*) *in the price war between providers.*

- *If* $\alpha \leq \frac{C_2}{C_1+C_2}$, *then* $p_1^* = v\left(\frac{C_1}{1-\alpha}\right) \geq p_2^* = v\left(\frac{C_2}{\alpha}\right)$. *The common zone is left to provider* 2 *by provider* 1.
- *If* $\alpha > \frac{C_2}{C_1+C_2}$ *then* $p_1^* = p_2^* = p^* = v(C_1 + C_2)$. *The common zone is shared by the providers.*

(Darker=more expensive)



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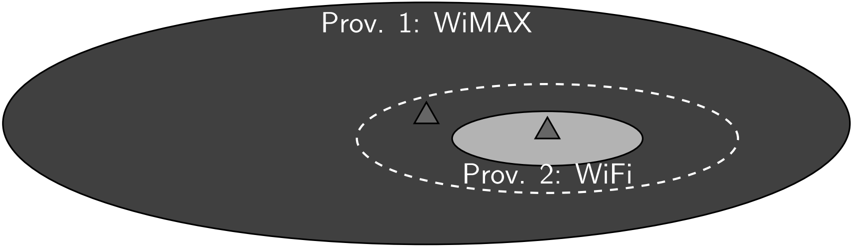

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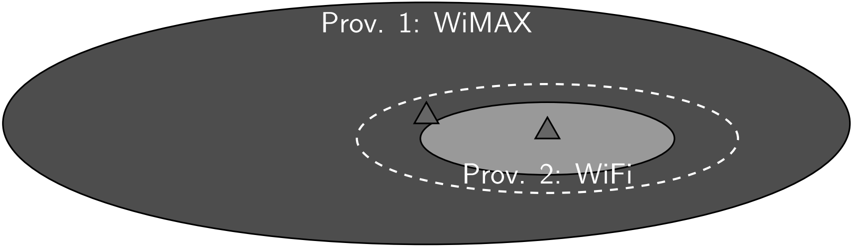

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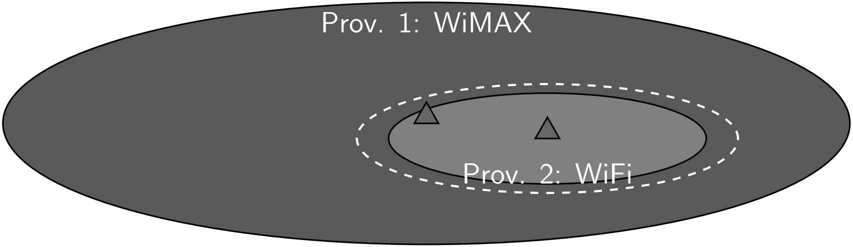

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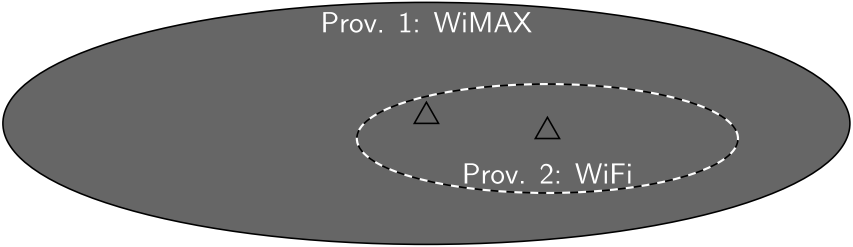

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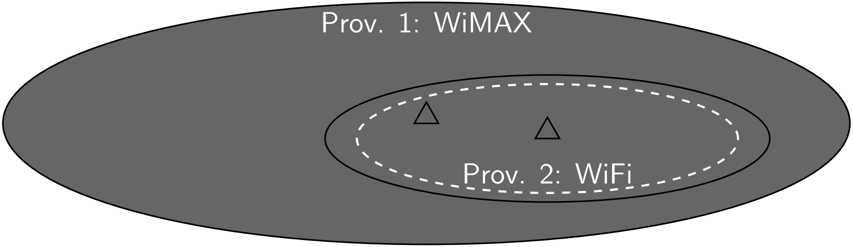

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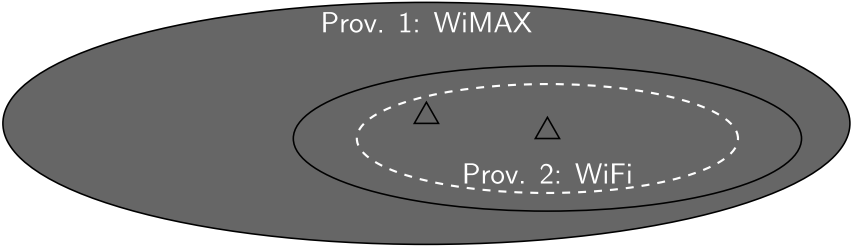
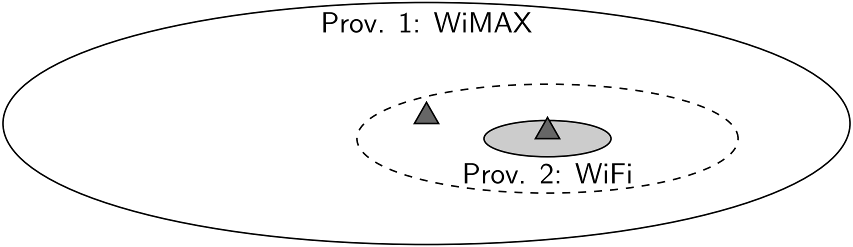


Optimization of transmission power

Consider **Provider 2 modifying his transmission power** (and thus his coverage area, still assumed in the competitor's coverage area)

Transmission power affects the proportion α of population covered.



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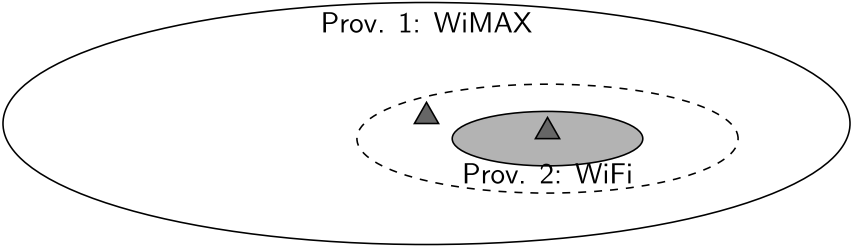

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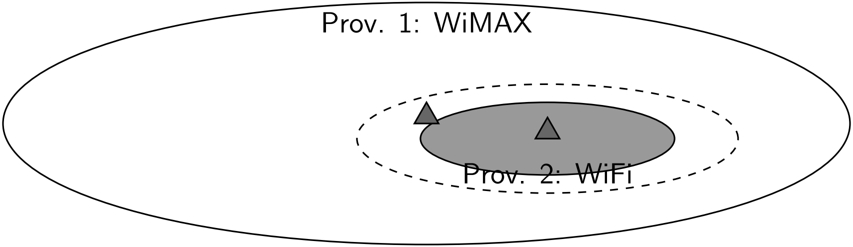

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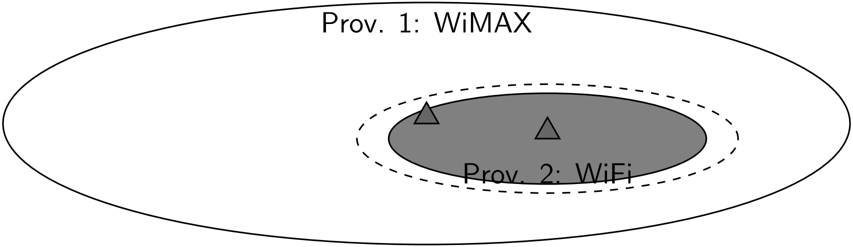

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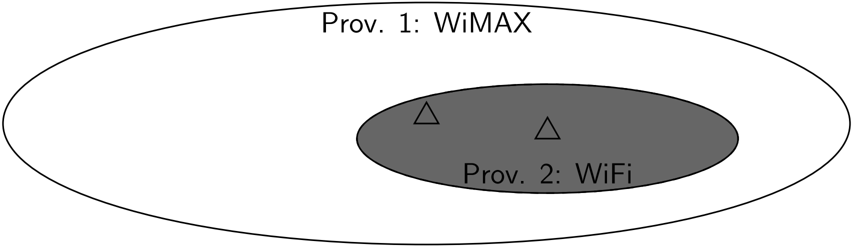

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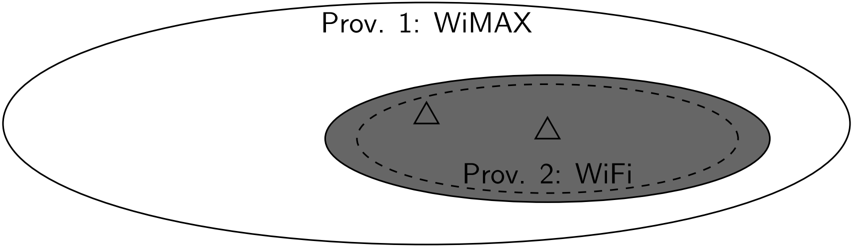

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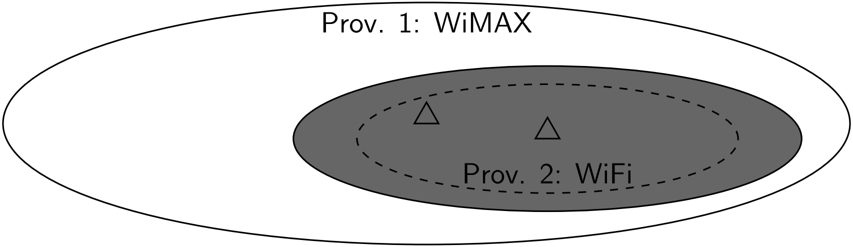

Revenue for provider 2 when transmission power varies

Assumption: **sequential decisions**

1. Provider 2 chooses α
2. Both providers play the pricing game (Nash equilibrium)

Revenue for provider 2 when transmission power varies

Assumption: **sequential decisions**

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2. Both providers play the pricing game (Nash equilibrium)

For a given α, that might imply a cost $\mathrm{Cost}_2(\alpha)$, we have at the Nash equilibrium of the pricing game

$$R_2(\alpha) = C_2 \times v\left(\max(C_2/\alpha, C_1 + C_2)\right) - \mathrm{Cost}_2(\alpha)$$

(recall that v=marginal valuation function, decreasing)

Determining the best α

Example 2: consider a simple model:

- signal attenuation of the form $c/\text{distance}^{\mu}$, with μ generally in $[2,5]$
- minimum reception power P_{min} to be covered by provider 2
- uniform repartition of population (so that $\alpha = \frac{\text{area covered by prov. 2}}{\text{area covered by prov. 1}}$)
- unit cost β for transmission power

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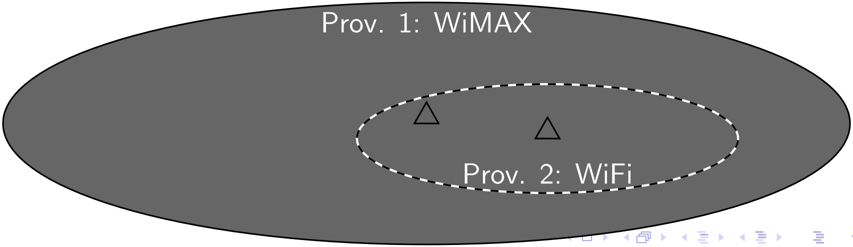
Then,

- if $-v'(C_1 + C_2) \geq \frac{\beta\mu P_{\text{min}}}{2c} C_2^{\mu/2-1}(C_1 + C_2)^{-\mu/2-1}$, then $\alpha^* = \frac{C_2}{C_1+C_2}$ and all users perceive the same price at equilibrium;



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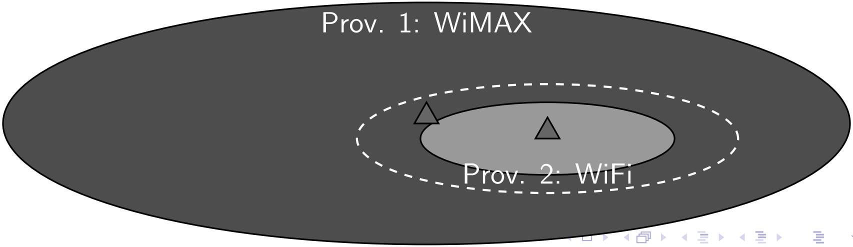
- otherwise $\alpha^* < \frac{C_2}{C_1+C_2}$, and provider 2 users experience a strictly lower price than users only covered by provider1.

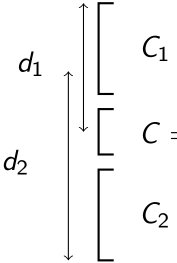


Outline

Partial spectrum sharing

Again one common coverage area and two providers, but **an amount C of spectrum has to be shared among providers**

- Each provider i still has some ??private" band C_i
- If $d_i > C_i$, demand in excess $d_i - C_i$ is sent to the shared band.
- The shared spectrum is allocated in proportion with the providers' excess demand



$$C = \begin{cases} C_1' = \frac{[d_1 - C_1]^+}{[d_1 - C_1]^+ + [d_2 - C_2]^+} C \\ C_2' = \frac{[d_2 - C_2]^+}{[d_1 - C_1]^+ + [d_2 - C_2]^+} C \end{cases}$$

User equilibrium characterization

User equilibrium characterization

$$\bar{p}_1 = p_1 \max\left(1, \frac{d_1}{C_1 + \frac{[d_1 - C_1]^+}{[d_1 - C_1]^+ + [d_2 - C_2]^+} C}\right)$$

$$\bar{p}_2 = p_2 \max\left(1, \frac{d_2}{C_2 + \frac{[d_2 - C_2]^+}{[d_1 - C_1]^+ + [d_2 - C_2]^+} C}\right)$$

Perceived prices depend on demands.

User equilibrium characterization

$$
\begin{aligned}
\bar{p}_1 &= p_1 \max\left(1, \frac{d_1}{C_1 + \frac{[d_1-C_1]^+}{[d_1-C_1]^+ + [d_2-C_2]^+} C}\right) \\
\bar{p}_2 &= p_2 \max\left(1, \frac{d_2}{C_2 + \frac{[d_2-C_2]^+}{[d_1-C_1]^+ + [d_2-C_2]^+} C}\right) \\
d_1 + d_2 &= D(\min(\bar{p}_1, \bar{p}_2))
\end{aligned}
$$

Perceived prices depend on demands.
Demand w.r.t. perceived price.

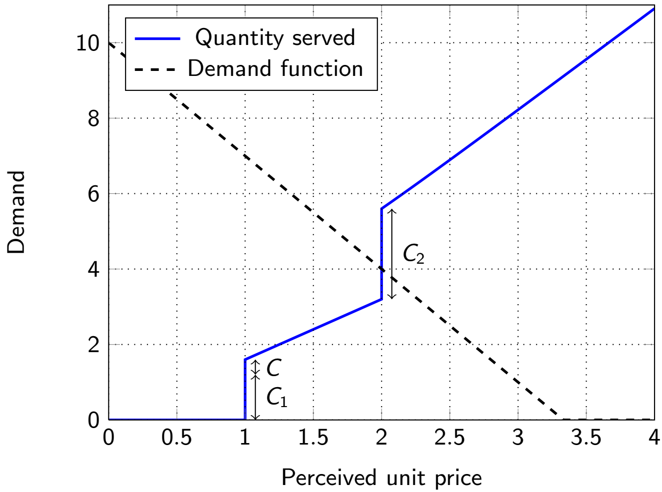
User equilibrium characterization

$$
\begin{aligned}
\bar{p}_1 &= p_1 \max\left(1, \frac{d_1}{C_1 + \frac{[d_1 - C_1]^+}{[d_1 - C_1]^+ + [d_2 - C_2]^+} C}\right) \\
\bar{p}_2 &= p_2 \max\left(1, \frac{d_2}{C_2 + \frac{[d_2 - C_2]^+}{[d_1 - C_1]^+ + [d_2 - C_2]^+} C}\right) \\
d_1 + d_2 &= D(\min(\bar{p}_1, \bar{p}_2)) \\
\bar{p}_1 > \bar{p}_2 &\Rightarrow d_1 = 0 \\
\bar{p}_2 > \bar{p}_1 &\Rightarrow d_2 = 0.
\end{aligned}
$$

Perceived prices depend on demands.
Demand w.r.t. perceived price.
Only cheapest providers get demand.

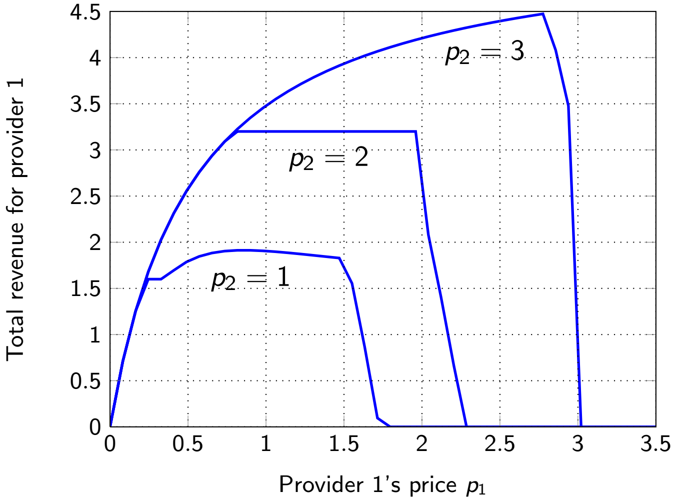
Proposition

Whatever the price profile (p_1, p_2), there exists at least one Wardrop equilibrium. The corresponding perceived prices are unique.

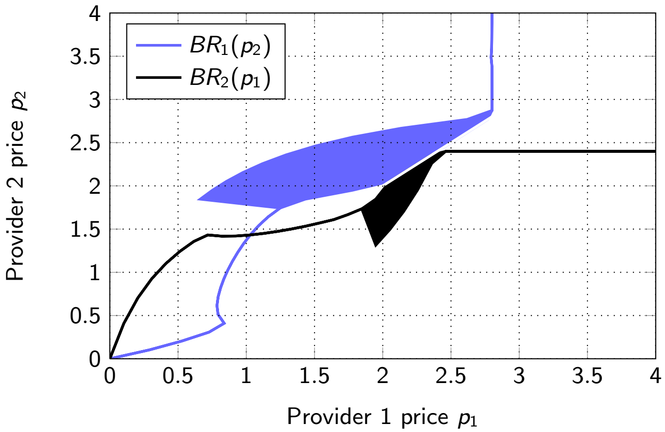


Provider utilities

$$R_i(p_1, p_2) := p_i d_i \text{ for } i \in \{1, 2\}.$$



Provider best-reply curves



Proposition

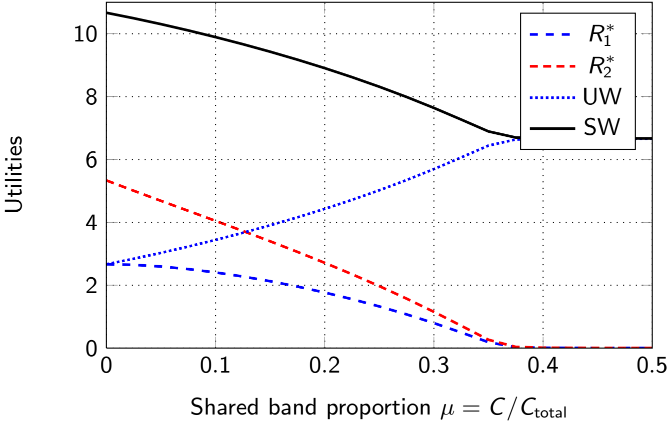
There is no Nash equilibrium without losses.

Social Welfare considerations

The Social Welfare at Nash equilibrium is

$$\mathsf{SW} = \min\left(1, \frac{C_1 + C_2 + C}{D(\bar{p})}\right) \int_0^{D(\bar{p})} v, \qquad (2)$$

Influence of the fraction μ of total available band that is unlicensed?



Outline

Conclusions and perspectives

We have analyzed some pricing games among providers

- Characterized how demand is split (following Wardrop's principle),
- studied the Nash equilibria of the pricing games (characterization, uniqueness),

for three specific situations:

- one common coverage area and dedicated bands
- two providers with dedicated bands, and imbricated coverage areas
- two providers with common coverage area and partially shared spectrum.

Perspectives

- Study of more complex topologies
- What if providers play on capacities along with prices?
- What about the dynamics of the model? How to drive to the equilibrium?

Thank you for your attention!