Budget-limited advertising game on sponsored search engines in competition. Impact on engine ranking.

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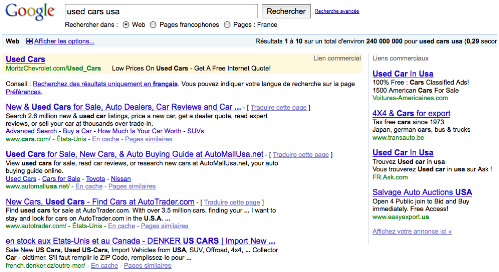
Outline

Introduction to adword auctions

- Search engines play a crucial role in the Internet.
- Revenue through advertising slots, usually displayed at the top or right of the search page.
- Advertisers submit bids for relevant keywords only.



- Allocation of slots thanks to adword auctions.
  - combined revenue of Yahoo! and Google in 2005: \$11 billion in 2005
  - expected to count for 40% of total advertising revenue.

Auction principle (single keyword, K slots)

- Advertisers submit bids for specific keywords.
- Each time there is a search on that keyword:
  - advertisers are ranked and allocated slots according to a prespecified criterion:
    - bid value (initially for Yahoo!)
    - the revenue they will generate (more or less Google).
  - Possible payment rules:
    - *Pay-Per-Impression* (PPI): advertisers charged every time their ad is displayed
    - *Pay-Per-Click* (PPC): advertisers is charged only when the ad is clicked
    - *Pay-Per-Transaction* (PPT): advertisers charged when a sell.
  - Amount to be paid each time?
    - First Price: advertisers pay their bid
    - Generalized Second Price (GSP): they pay the bid of advertiser below them in the ranking
    - Vickrey-Clarke-Groves (VCG) auctions: you pay the opportunity cost that your presence introduce to all other advertisers.
- In use: PPC and GSP. Bid-based and revenue-based rankings implemented.

Goal of our work

- Almost all works deal with monopolistic search engines,
- But those engines are in competition
- We present two models
- What is the best (limited-)budget repartition between the engines for advertisers (both models)?
- What is the best ranking strategy given this competition (second model)?

  ⇒ Non-cooperative game between search engines.
  - Two-levels game:
    - Largest time scale: search egines choose their ranking strategy (maximizing) their revenue
    - Smallest time scale: advertisers in competition for the advertising slots (by splitting their avertisement budget over the engines)
  - Game played by backward induction.
- Both models require to compute the average prices and winning probabilities at search engines depending on the strategies of all actors.

Outline

Basic model

- Two search engines (SE), labelled 1 and 2
  - a single advertisement slot at each SE
  - considering a single keyword, with λ, average number of searches per unit of time
  - α: (fixed) market share of SE 1 ($\alpha\lambda$ searches on SE 1).
  - Both SE apply GSP charging scheme.
- k advertisers:
  - limited budget b, fixed (1st model) or taken from cdf $G(b)$ (second model)
  - valuation per click v, taken from the cdf $F(v)$
  - Click-Through-Rate (CTR) *separable*, as the product of the CTR of the search engine, q_1 and q_2 for SE 1 and 2 respectively, and of the CTR c_i of the considered Advertiser i.
  - Goal of advertiser i: split their budget b_i over the two SEs: β_i on SE 1 and $1 - \beta_i$ on SE 2.
  - Remark: advertisers' interest is to bid their valuation v_i since GSP is VCG when a single slot.

Outline

Additional assumptions, definitions, and properties

- All advertisers know the total number K of advertisers interested in the considered keyword
- All advertisers have the same budget limit b.
- All advertisers have the same CTR c (bid-based=revenue-based).
- Advertisers will try to deduce the valuations of opponents from their bidding probability
- Definition: function $p_i = p(v_i)$ bidding probability depending on valuation v_i.
- Goal: which function $p(v)$ constitutes a Nash equilibrium of the bidding game, i.e., st bidding $p(v_i)$ is the best strategy $\forall i$?

Results

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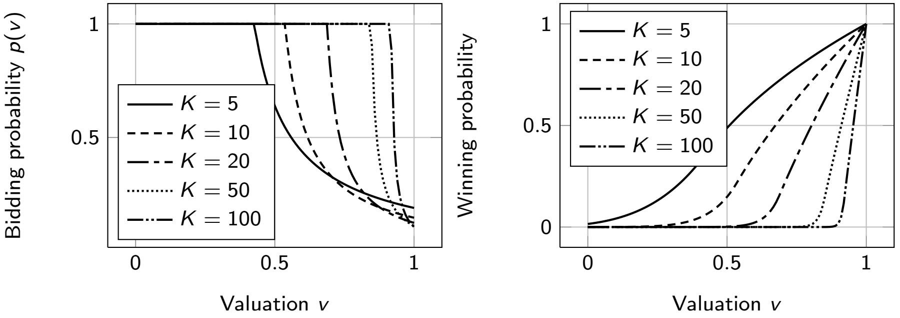
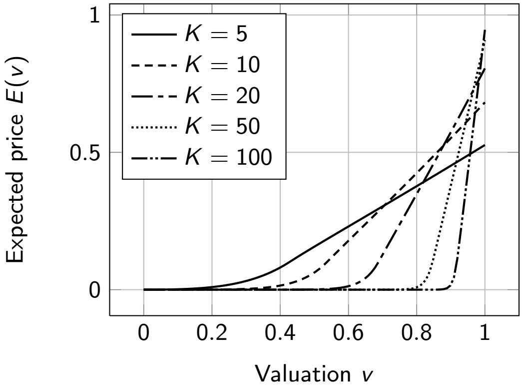
- Assuming that the valuation distribution F admits a density f and has a finite mass, i.e. $\int f(v)dv < +\infty$, there exists an equilibrium bidding strategy function $p(\cdot)$.
- Numerical examples: F Uniform over [0,1]:



- low-valuation advertisers bid more often (less likely to win)
- below a level: participate to all auctions (budget not fully spent)
- bidding probability not monotone in K: trade-off between winning prob. due to price to pay if winning.

Price $E(v)$ that an advertiser expects to pay when he bids v



- Low-valuation advertisers pay less per auction when the number of competitors increases (since they have less chances to win the auction),
- while high-valuation advertisers will pay more (they frequently win the auctions they participate in, but they face higher bids).

Budget repartition between two search engines

- Two search engines (e.g., Google and Yahoo!)
- Which budget repartition of advertisers between those two search engines ?
- Still assuming all advertisers are assumed to have the same total budget b, and the same click-through rate q_ℓ on ℓ.
- An advertiser decide, depending on his valuation, how to spread that budget between the two search engines.
- Equivalent to determining the bidding probability p_ℓ on each search engine ℓ.
- The equilibrium $p_\ell(v)$ can be determined numerically.

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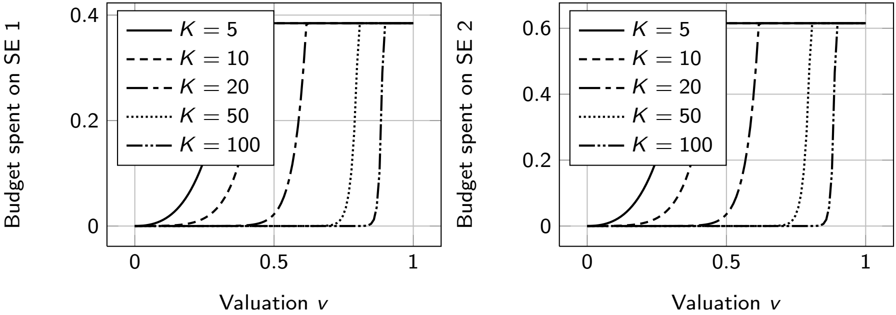
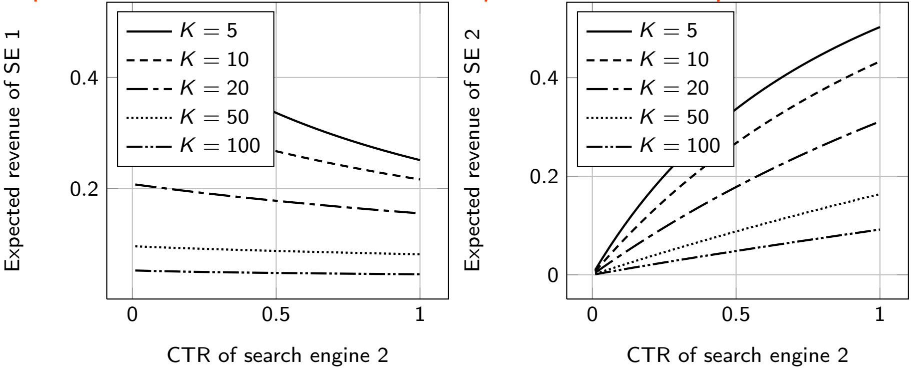
Numerical results



- Search engines with different CTR ($q_1 = 0.8$ and $q_2 = 0.5$):
- the budgets spent by advertisers on both search engines proportional to the ratio of CTRs (in accordance with the theory).

Expected revenues of SEs with $q_1 = 0.5$ while q_2 varies



- The total revenue of search engine 2 appears to be a concave nondecreasing function of its CTR.
- Also a concave nondecreasing function of its user request frequency (only sensitive to the product request_frequency×CTR).

⇒ anticipating the advertiser reaction should be taken into account to make the optimal investment decisions.

Outline

Second model: different assumptions

- Not the same budget for all advertisers, independently taken from cdf $G(b)$, and different CTRs.
- Advertisers also
  - know their own bid
  - know the distribution of valuations and budgets of other players
  - observe their bidding probability.
  - do not try to deduce the valuation and budget of competitors from their bidding probabilities.
- With this: able
  - to derive results for both revenue-based and bid-based ranking
  - and to play an upper-level game on raning between search engines.

Key parameters and advertisers utilities

- Key performance parameters:
  - $w_j(v_i)$: probability that i wins on SE j given that her bid/valuation is v_i
  - $\mathbb{E}[R_j|v]$ being the average price paid on SE j *having won with* v.
  - $p_i^{(j)}$: probability that advertiser i submits a bid on SE j.
- Those parameters can be computed for both bid-based and revenue-based rankings, by solving sets of equations.
- Revenues, for $\beta = (\beta_1, \ldots, \beta_k)$ profile of strategies of advertisers:

$$U_i(\beta) = q_1 c_i w_1(v_i) \lambda_i^{(1)} (v_i - \mathbb{E}[R_1|v_i]) q_2 c_i w_2(v_i) \lambda_i^{(2)} (v_i - \mathbb{E}[R_2|v_i]).$$

Game between advertisers on budget repartition at each SE

We consider two SEs and two advertisers.

- The two advertisers play, trying to (selfishly) maximize their own utility/revenue.
- Equilibrium notion, Nash equilibrium: profile of proportion strategies (β_1^*, β_2^*) st $\forall \beta_1, \beta_2 \in [0,1]$,

$$U_1(\beta_1^*, \beta_2^*) \geq U_1(\beta_1, \beta_2^*) \text{ and } U_2(\beta_1^*, \beta_2^*) \geq U_2(\beta_1^*, \beta_2).$$

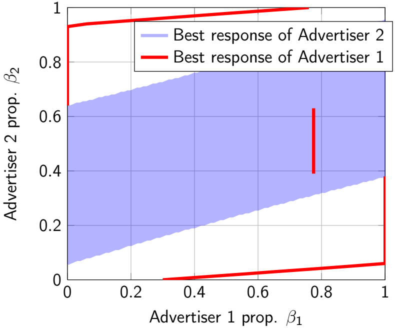
- To determine the Nash equilibria (if any), we define the *best response* of each advertiser as a function of the strategy of its opponent:

$$\begin{aligned} \mathrm{BR}_1(\beta_2) &= \arg\max_{\beta\in[0,1]} U_1(\beta, \beta_2) \text{ and} \\ \mathrm{BR}_2(\beta_1) &= \arg\max_{\beta\in[0,1]} U_2(\beta_1, \beta). \end{aligned}$$

- Graphically, the set of Nash equilibria is the (possibly empty) set of intersection points of BR curves.

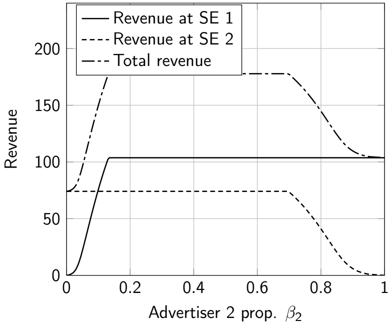
Illustration: both SEs implement bid-based pricing

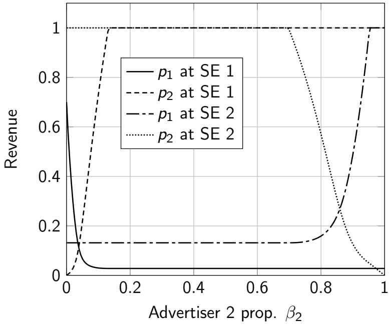
$V \equiv U[0,20]$, $\alpha = 0.6$, $\lambda = 100$, $q_1 = 0.5$, $q_2 = 0.6$, $b_1 = 5$, $c_1 = 0.5$, $b_2 = 20$, $c_2 = 0.4$, $v_1 = 10$, $v_2 = 9$ and $p_r = 0.1$



- for each fixed β_1, there is actually an interval for the best response of Advertiser 2 (blue)
- best response of Advertiser 1 in terms of β_2, we obtain the red curve
- set of Nash equilibria: $\{0.775\} \times [0.39, 0.63]$.

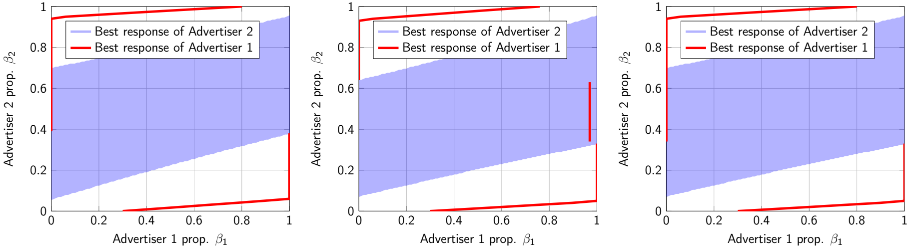
Explanation of an interval as best response, for $\beta_1 = 0.21$





- Left: revenue indeed constant
- Right: bidding probability constant
- The probability of bidding is maximal, equal to 1, independent of the submitted budget $\Rightarrow$ the budget is not fully spent.

3 other cases of ranking possibilities: bid based- revenue based (B-R), R-B and R-R



Nash equilibria:

- For the B-B case, all the profiles $(\beta_1, \beta_2) \in \{0.775\} \times [0.39, 0.63]$;
- for the B-R case, it is $\{0\} \times [0.39, 0.695]$;
- for the R-B case, $\{0.97\} \times [0.34, 0.63]$;
- for the R-R case, $\{0\} \times [0.34, 0.695]$.

Remark: when SE 2 implements revenue-based ranking, Advertiser 1's strategy at a Nash equilibrium is to put all her budget on SE 2.

Game between SEs on the ranking strategy

Anticipating the advertisers' decisions, the SEs seek to maximize their revenues from advertisement.
Another level of game: $(\text{Rev}_1, \text{Rev}_2)$ in terms of the rules used by SE 1 (line) and SE2 (column)

	B	R
B	$(10.15, 3.62)$	$(1.20, 11.06)$
R	$(11.32, 1.32)$	$(1.50, 11.06)$

- Best responses in red. Nash equilibrium: both elements in red.
- R-R: Nash equilibrium: in agreement with Yahoo!'s move to follow Google.
- For other sets of parameters such that B-R is an equilibrium.
  $\Rightarrow$ close look necessary for SEs!

Outline

Conclusions/Future activities

We have

- defined two model describing two search engines in competition for advertisers in adword auctions
- derived how advertisers should behave depending on the ranking strategies of engines
- llustrated how engines can (competitively) play on the ranking strategy.

Future activities:

- extend our study to the situation where SEs propose more than one slot
- look more closely at Google against Yahoo!