A short introduction to adword auctions and the advantage of introducing randomness

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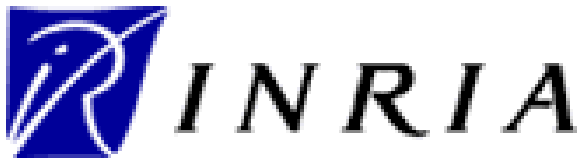

Outline

1. Introduction to adword auctions
2. Basic auction principle
3. Model and analysis with a random slot assignment
4. VCG auction between advertisers for displaying probability
5. Comparison with deterministic GSP
6. Future activities

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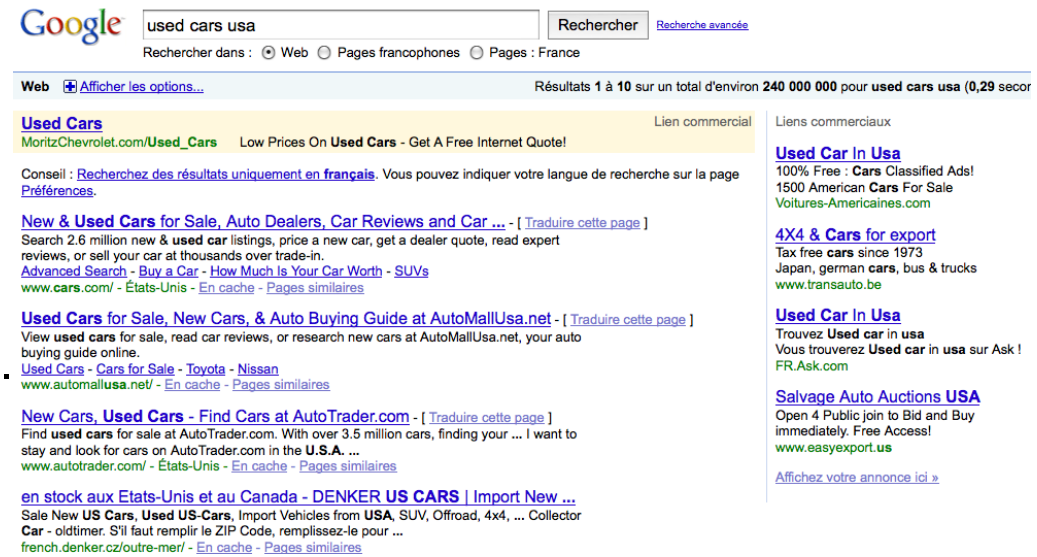
Introduction to adword auctions

- Search engines play a crucial role in the Internet.
- Revenue through advertising slots, usually displayed at the top or right of the search page.
- Advertisers submit bids for relevant keywords only.



- Allocation of slots thanks to adword auctions.
  - combined revenue of Yahoo! and Google in 2005: $11 billion in 2005
  - expected to count for 40% of total advertising revenue.

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Auction principle (single keyword, K slots)

- Advertisers submit bids for specific keywords.
- Each time there is a search on that keyword:
  - advertisers are ranked and allocated slots according to a prespecified criterion:
    - ★ bid value (initially for Yahoo!)
    - ★ the revenue they will generate (more or less Google), taking into account the (learned) click-through rate (CTR).
  - Possible payment rules:
    - ★ *Pay-Per-Impression* (PPI): advertisers charged every time their ad is displayed
    - ★ *Pay-Per-Click* (PPC): advertisers is charged only when the ad is clicked
    - ★ *Pay-Per-Transaction* (PPT): advertisers charged when the click results in a real sell.
  - Amount to be paid each time?
    - ★ First Price: advertisers pay their bid
    - ★ Generalized Second Price (GSP): they pay the bid of advertiser below them in the ranking
    - ★ Vickrey-Clarke-Groves (VCG) auctions: you pay the opportunity cost that your presence introduce to all other advertisers.
- In use: PPC and GSP.

Few current or important research issues in adword auctions

- Incentive compatibility (truthful bidding) of the GSP pricing scheme if game not one-shot and advertisers can anticipate strategies of competitors.
- Budget limit of advertisers and consequences on their strategies.
- CTR assumed in the simplest case to depend on the advertiser relevance and slot position. But it may also depend on other allocated advertisers.
  - Several models for a better characterization
- What about competition between search engines?
  - Which allocation rule to apply in that context?
  - which payment rule?

A specific issue: users may compose the same keyword several times

- This situation may happen when users
  - do not remember the results
  - or require new or additional informations
- Traditional adword auctions will *always* display the same advertisers.
- But is it the most relevant procedure ?
  - If the ad not clicked through once, why always presenting it again?
- We propose to illustrate the potential benefits of using a

random allocation rule.

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Model

Simplified model to illustrate the phenomenon.

- A search engine providing only one commercial slot
- Two advertisers, say, 1 and 2, competing for that slot on a given adword:
  - b_i bid of advertiser i for that keyword,
  - π_i the probability that advertiser i's ad is displayed (which should depend on the bid profile (b_1, b_2)),
  - p_i the price-per-click that advertiser i is charged, also dependent on (b_1, b_2).

In a first step, we fix the bids and prices, and investigate the gain produced by a random assignment.

User behavior model

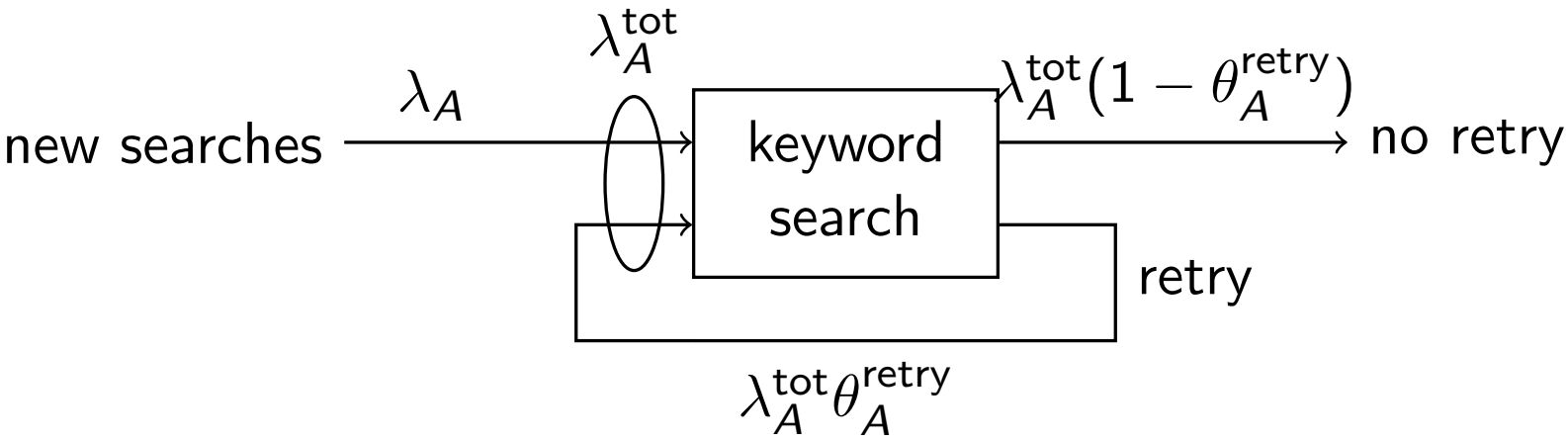
- Heterogeneous population:
  - type-A users only interested in purchasing the good sold by advertiser 1, but can potentially click on the ad of advertiser 2 without purchasing it eventually.
  - type-B users behaving symmetrically with respect to advertiser 2.
- λ_A (resp. λ_B) average number of *first* requests per time unit of type-A (resp. type-B) customers.
- Click propability of a type-A (resp. B) customer if ad of advertiser i displayed: $c_{A,i}$ (resp. $c_{B,i}$).
- Purchasing probabilities $h_{A,1}, h_{B,2} > 0$ after clicking on the (corresponding) ad.
- Buying probabilities respectively for type-A and type-B customers:

$$\pi_1 c_{A,1} h_{A,1} \quad \text{and} \quad \pi_2 c_{B,2} h_{B,2}.$$

Customers not buying will retry later

- R_j probability of a not-buying type-$j \in \{A, B\}$ user to perform again the search later.
- For type-A: θ_A^{retry} the overall probability of retry, given by

$$\theta_A^{\text{retry}} := R_A(1 - \pi_1 c_{A,1} h_{A,1}).$$



- From $\lambda_A^{\text{tot}} = \lambda_A + \theta_A^{\text{retry}} \lambda_A^{\text{tot}}$,

$$\lambda_A^{\text{tot}} = \frac{\lambda_A}{1 - \theta_A^{\text{retry}}} = \frac{\lambda_A}{1 - R_A(1 - \pi_1 c_{A,1} h_{A,1})}.$$

- Similarly, $\lambda_B^{\text{tot}} = \frac{\lambda_B}{1 - R_B(1 - \pi_2 c_{B,2} h_{B,2})}$.

Search engine revenue

- Mean revenue per unit of time (with $\pi_1 + \pi_2 = 1$) :

$$\begin{aligned} U(\pi_1) &= p_1 \underbrace{\pi_1(\lambda_A^{\text{tot}} c_{A,1} + \lambda_B^{\text{tot}} c_{B,1})}_{\text{nb of clicks on ad 1}} + p_2 \underbrace{\pi_2(\lambda_B^{\text{tot}} c_{B,2} + \lambda_A^{\text{tot}} c_{A,2})}_{\text{nb of clicks on ad 2}} \\ &= \lambda_A \frac{\pi_1(p_1 c_{A,1} - p_2 c_{A,2}) + p_2 c_{A,2}}{1 - R_A(1 - \pi_1 c_{A,1} h_{A,1})} + \lambda_B \frac{\pi_1(p_1 c_{B,1} - p_2 c_{B,2}) + p_2 c_{B,2}}{1 - R_B(1 - c_{B,2} h_{B,2} + \pi_1 c_{B,2} h_{B,2})} \end{aligned}$$

- **Proposition:** there exists a unique π_1^* maximizing the revenue $U(\pi_1)$ of the search engine. The solution is in the interior of the interval $[0, 1]$ if $U'(0) > 0$ and $U'(1) < 0$.

Symmetric case: $c_{A,1} = c_{B,2} = c$, $h_{A,1} = h_{A,2} = h$, $R_A = R_B = R$

Assuming also $c_{A,2} = c_{B,1} = 0$ (users only click on "their" advertiser)

- **Proposition:** The value π_1^* of π_1 maximizing the search engine revenue is
  - $\pi_1^* = 0$ if $\sqrt{\frac{\lambda_A p_1}{\lambda_B p_2}} \leq \frac{1-R}{1-R(1-ch)}$,
  - $\pi_1^* = 1$ if $\sqrt{\frac{\lambda_B p_2}{\lambda_A p_1}} \leq \frac{1-R}{1-R(1-ch)}$,
  - $\pi_1^* = \frac{1}{R \cdot ch} \frac{1-R+R\cdot ch-(1-R)\sqrt{\frac{\lambda_B p_2}{\lambda_A p_1}}}{\sqrt{\frac{\lambda_B p_2}{\lambda_1 p_1}}+1}$ if $\frac{1-R}{1-R(1-ch)} < \sqrt{\frac{\lambda_A p_1}{\lambda_B p_2}} < \frac{1-R(1-ch)}{1-R}$.
- **Interpretation**: when R close to 1, the search engine gets a higher revenue with a random policy.
- **Example:** $c = 1/2$, $h = 1/2$, $\lambda_A = 1$, $\lambda_B = 0.8$, $p_1 = 1$, $p_2 = 0.8$.
  - With retry probability $R = 0.8$, the revenue is maximized at $\pi_1^* = 2/3$, and given by 1.4.
  - Compared with the optimal revenue when only one ad is displayed, $\max(\lambda_A p_1, \lambda_B p_2) = 1$, a gain of 40% is observed.

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Game between advertisers

- The previous analysis was for fixed bids and prices. But advertisers can play (i.e., submit bids representing how much they accept to pay) strategically according to the display probability.
- In the symmetric case (the asymmetric case handled similarly), mean sale incomes per unit of time (if v_i benefit per sale):

$$V_1(\pi_1) = \underbrace{\lambda_A^{\text{tot}} \pi_1 ch}_{\text{sales per time unit}} \quad v_1 = \frac{\lambda_A \pi_1 ch}{1 - R(1 - \pi_1 ch)} v_1$$

$$V_2(\pi_2) = \frac{\lambda_B \pi_2 ch}{1 - R(1 - \pi_2 ch)} v_2.$$

- Utilities

$$U_i = V_i(\pi_i) - p_i.$$

VCG auction

- Mechanism known to be *incentive compatible*, *individually rational* and *efficient* by maximizing social welfare.
- **Allocation rule** from the search engine: solve strictly convex optimization problem

$$\max_{\pi_1,\pi_2 \text{ s.t. } \pi_1+\pi_2\leq 1} \bar{V}_1(\pi_1) + \bar{V}_2(\pi_2),$$

  where $\bar{V}_i$ is the declared willingness-to-pay function of advertiser i ($\bar{V}_i = V_i$ if i bids truthfully).
- Solution known from the previous propositions.

VCG pricing rule

- Charged the loss of value each advertiser imposes on the other through its presence.
- Total price t_i per time unit that each advertiser i is charged under the VCG rule is given by

$$\begin{cases} t_1 = \bar{V}_2(1) - \bar{V}_2(\bar{\pi}_2) \\ t_2 = \bar{V}_1(1) - \bar{V}_1(\bar{\pi}_1). \end{cases}$$

- If converted as a price per click

$$p_1 = \left(\bar{V}_2(1) - \bar{V}_2(\bar{\pi}_2)\right) \frac{1 - R(1 - \bar{\pi}_1 ch)}{\lambda_A \bar{\pi}_1 c},$$

$$p_2 = \left(\bar{V}_1(1) - \bar{V}_1(\bar{\pi}_1)\right) \frac{1 - R(1 - \bar{\pi}_2 ch)}{\lambda_A \bar{\pi}_2 c}.$$

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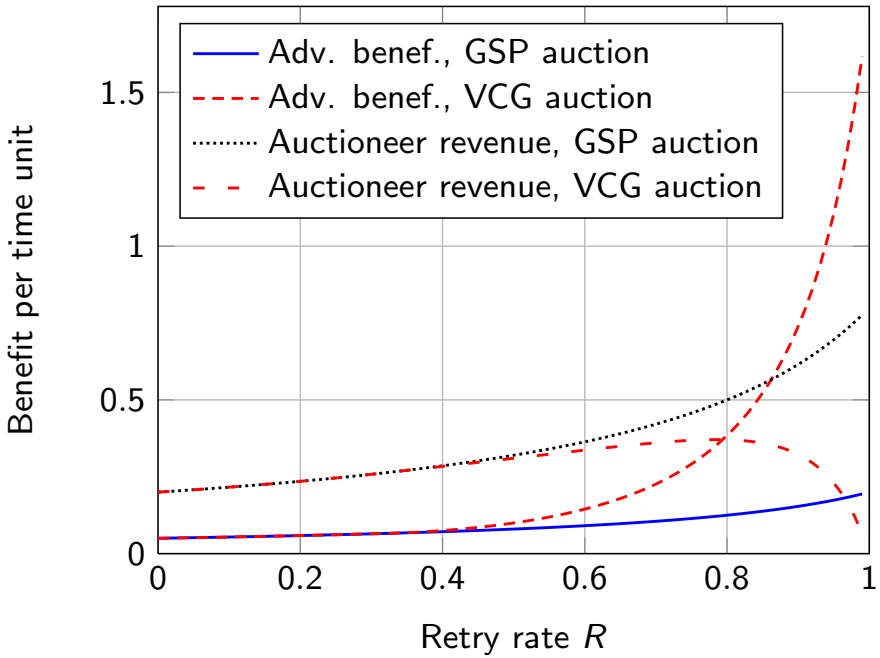
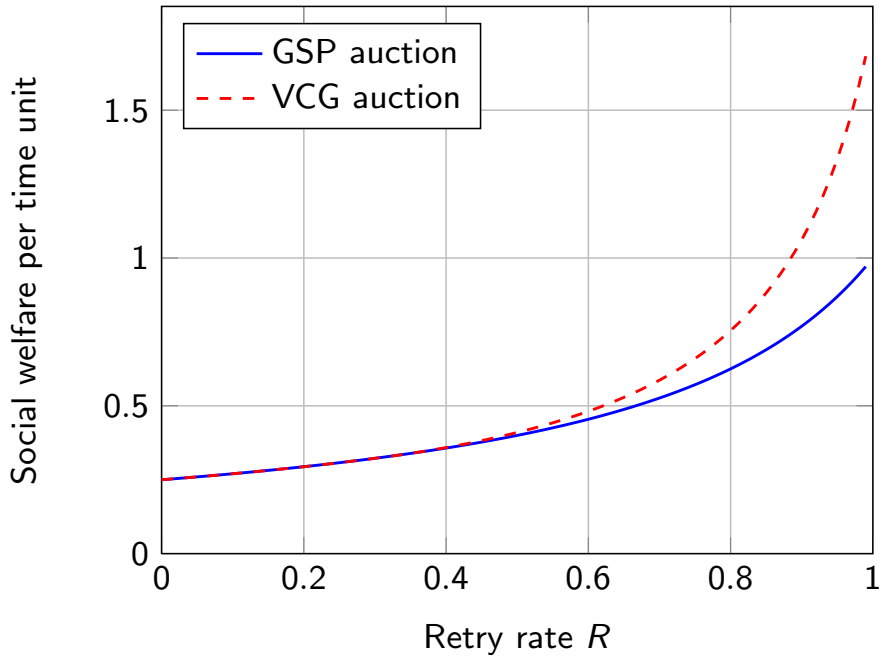
Comparison to deterministic GSP

Let $v_1 = 1$, $v_2 = 0.8$, $\lambda_A = \lambda_B = 1$, $c = h = 0.5$ and investigate the influence of R.





- Revenue of advertisers larger with VCG on display probability, but not the engine revenue
- In the case of competitive engines, the one applying VCG is likely to be prefered.
- Social welfare better with VCG and increases with R.

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Next activities on adword auctions

- Continuing our line of research on random allocation rule
  - Case of multiple slots
  - game between advertisers when GSP pricing rule with a reserve price is applied.
- More generally, working on competition among search engines:
  - Ex: model Google against Yahoo!
  - Other games for different engines with different search rates and/or rules: survival under competition?
  - Investigate in general a game on allocation and pricing *strategies*.